Reference textbook – James Stewart, Calculus 6th ed., Thomson, 2009

13016103 Mathematics 3

#4 Vector Functions, Part I

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Content of this lecture

- Vector functions and space curves

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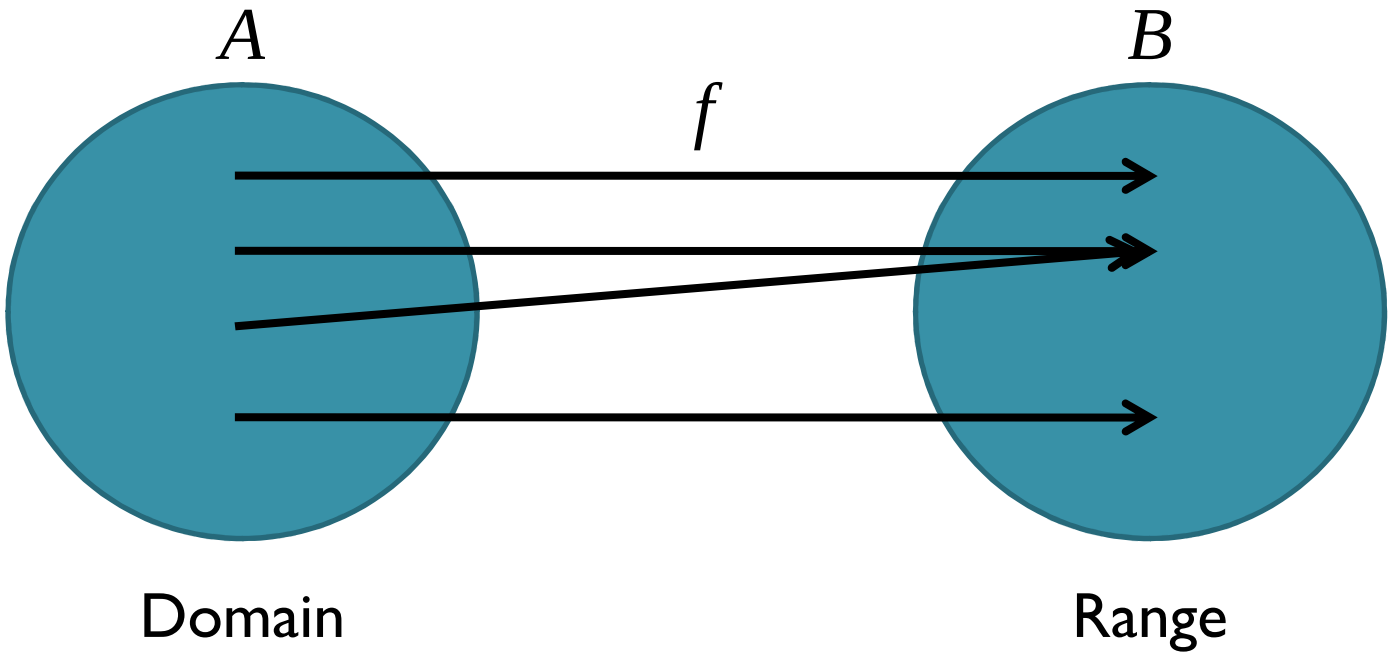
VECTOR FUNCTIONS AND SPACE CURVES

What is a function?

- In general, a function is a rule that assigns to each element in the domain an element in the range.

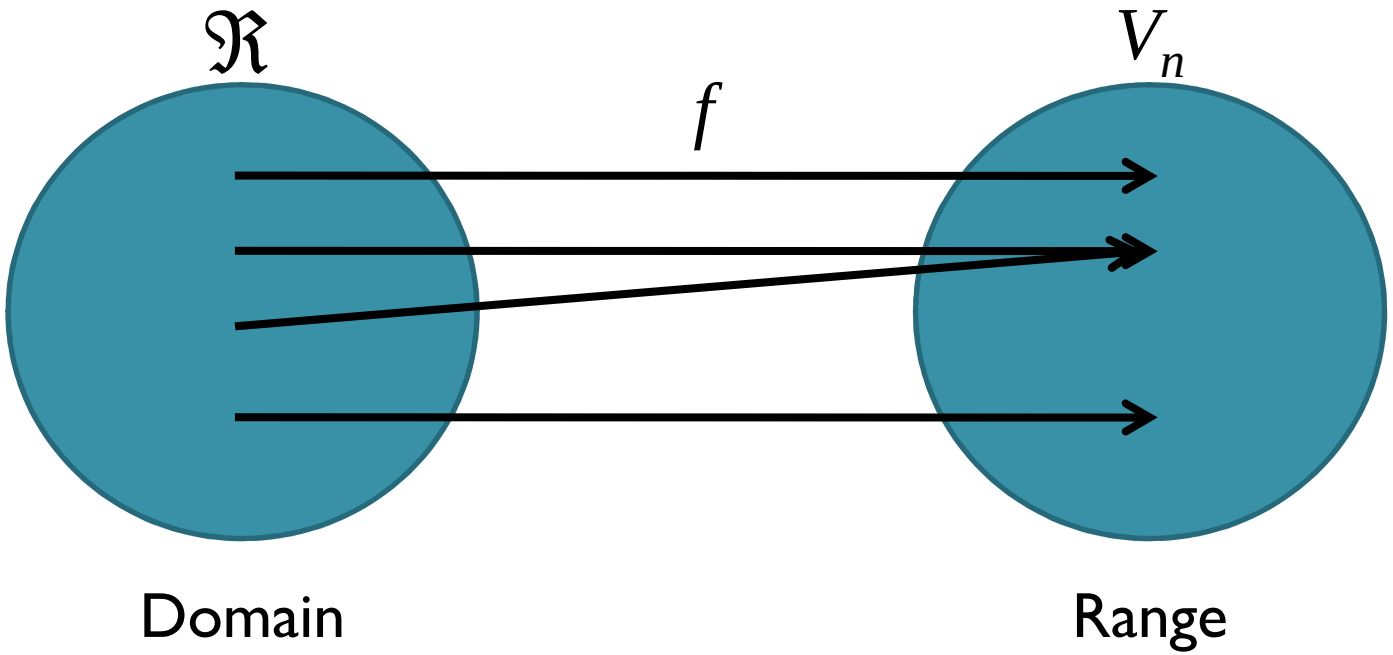


What is a function?

- So far, the functions that we have been using have been real-valued functions (e.g., $y = f(x)$).
- We now study functions whose values are vectors.
- Such functions are needed to describe curves and surfaces in space.

Vector functions

- A *vector-valued function*, or *vector function*, is simply a function whose domain is a set of real numbers and whose range is a set of vectors.



Vector functions

- We are most interested in vector function $\mathbf{r}$ whose values are three-dimensional vectors.
- This means that for every number t in the domain of $\mathbf{r}$, there is a unique vector in V_3 denoted by $\mathbf{r}(t)$.
- If $f(t)$, $g(t)$, and $h(t)$ are the components of the vector $\mathbf{r}(t)$, then f, g, and h are real-valued functions called the component functions of $\mathbf{r}$ and we can write

$$\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle = f(t)\,\mathbf{i} + g(t)\,\mathbf{j} + h(t)\,\mathbf{k}$$

Example 1

- If $\mathbf{r}(t) = \langle t^3, \ln(3 - t), \sqrt{t} \rangle$
  then the component functions are

$$f(t) = t^3 \qquad g(t) = \ln(3 - t) \qquad h(t) = \sqrt{t}$$

- The domain of $\mathbf{r}$ consists of all values of t for which the expression $\mathbf{r}(t)$ is defined.
- In this case, $3 - t \geq 0$ and $t \geq 0$.
- Therefore, the domain of $\mathbf{r}$ is the interval $[0, 3)$

Limit of a vector function

- The limit of a vector function r is defined by taking limits of its component functions as follows:

1 If $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$, then

$$\lim_{t \to a} \mathbf{r}(t) = \left\langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \right\rangle$$

provided the limits of the component functions exist.

- Note that limits of vector functions obey the same rules as limits of real-valued functions.

Limit of a vector function

- A vector function $\mathbf{r}$ is continuous at a if

$$\lim_{t \to a} \mathbf{r}(t) = \mathbf{r}(a)$$

- From the previous definition of limit, this means that $\mathbf{r}$ is continuous at a if and only if its component function f, g, and h are continuous at a.

Example 2

- Find $\lim_{t \to 0} \mathbf{r}(t)$, where

$$\mathbf{r}(t) = (1 + t^3)\,\mathbf{i} + te^{-t}\,\mathbf{j} + \frac{\sin t}{t}\,\mathbf{k}.$$

Space curve

- Suppose that f, g, and h are continuous real-valued functions on an interval I.
- Then the set C of all points (x, y, z) in space, where

$$x = f(t) \qquad y = g(t) \qquad z = h(t)$$

and t varies throughout the interval I, is called a *space curve*.

- The above equations are called *parametric equations of* C and t is called a *parameter*.

Example 3

- Describe the curve defined by the vector function

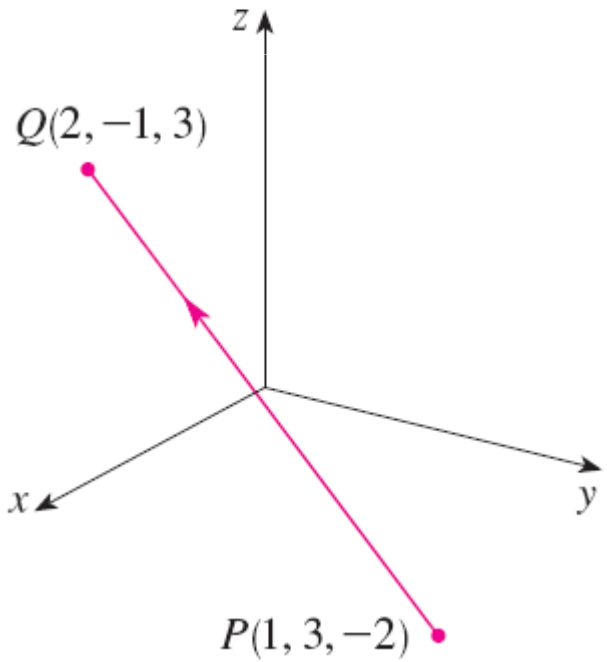
$$\mathbf{r}(t) = \langle 1 + t, 2 + 5t, -1 + 6t \rangle$$

Example 4

- Sketch the curve whose vector equation is $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t\,\mathbf{k}$

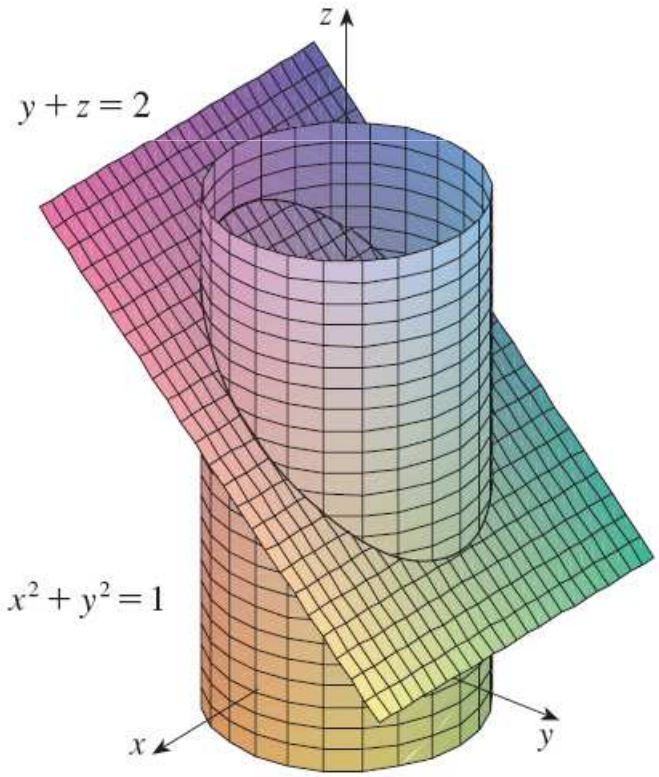
Example 5

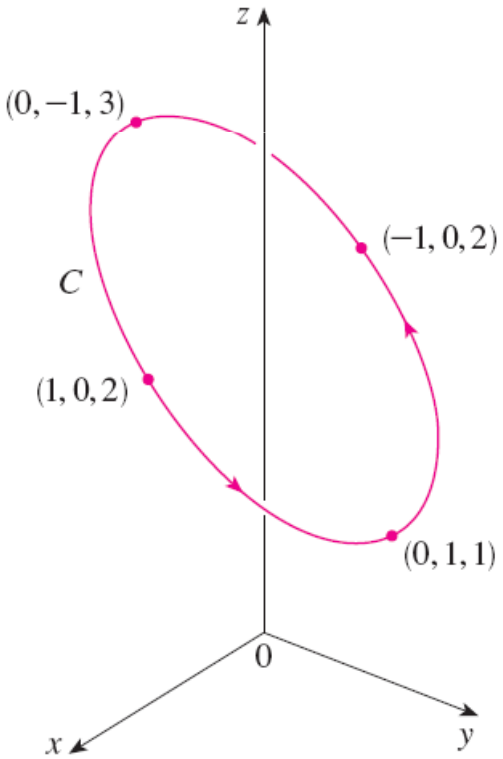
- Find a vector equation and parametric equations for the line segment that joins the point *P(1, 3, -2)* to the point *Q(2, -1, 3)*.



Example 6

- Find a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the plane $y + z = 2$.





Use computer to draw space curves

- Space curves are inherently more difficult to draw by hand than plane curves.
- Sometimes, we need a computer device to plot space curves.

$$x = (4 + \sin 20t)\cos t \qquad y = (4 + \sin 20t)\sin t \qquad z = \cos 20t$$

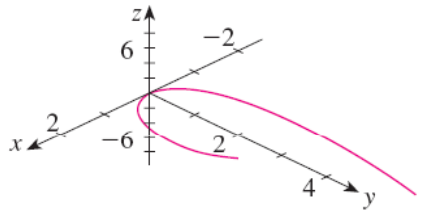
Use computer to draw space curves

- Space curves are inherently more difficult to draw by hand than plane curves.
- Sometimes, we need a computer device to plot space curves.

$$x = (2 + \cos 1.5t) \cos t \qquad y = (2 + \cos 1.5t) \sin t \qquad z = \sin 1.5t$$

Example 7

- Use a computer device to draw the curve with vector equation $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$.



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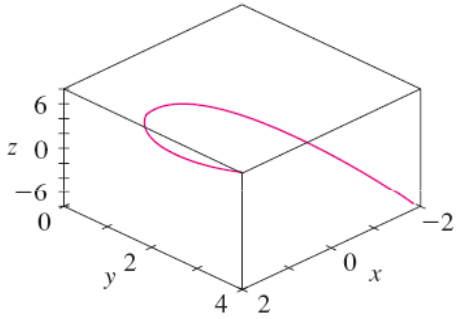

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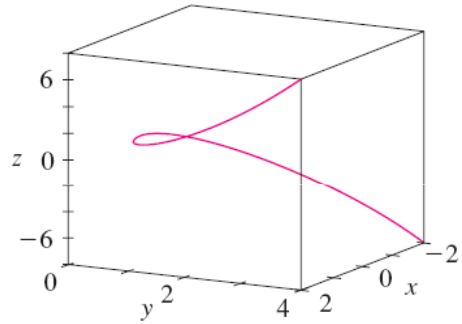

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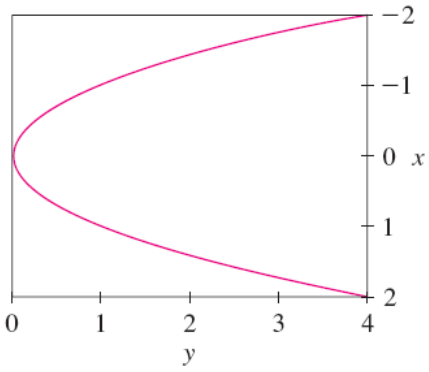

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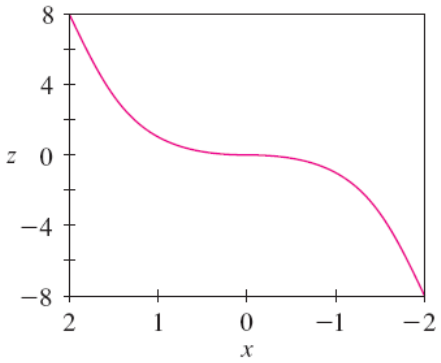

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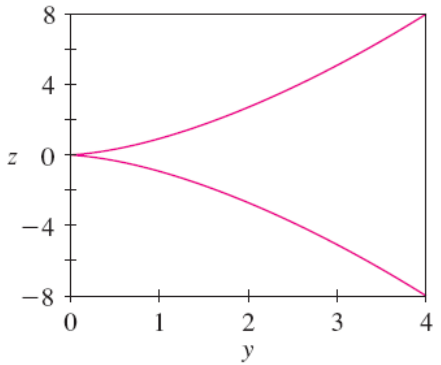

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